Gifted Students Education in China — Introduction of Chinese Mathematical Competitions

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ABSTRACT The development of Chinese mathematics competition activities can be divided into the following stages: Stage I (1956–1964): Considered as the birth of China's competition mathematics, Stage II (1978–1984): A working committee was set up by the Chinese Mathematical Society (CMS) in order to standardize and institutionalize the development of mathematics competition activities in China, Stage III (1985–present): This period sees a flourish of mathematics competition activities. China began to participate in the International Mathematical Olympiad (IMO) and in 1990, the 31st IMO was successfully held in Beijing on an unprecedented scale.

After more than three decades' exploration and practice, an ever-enriching, relatively stable system has evolved in the Chinese high school mathematical Olympiad practice.

Competition mathematics education is beneficial to the development of gifted students' mathematical ability in various aspects.

Competition mathematics was introduced to China in 1956 (cf. Hua, 1956a, 1956b). In the same year, mathematics competitions for senior high school students were held in Beijing, Tianjin, Shanghai and Wuhan respectively. Luogeng Hua (known as Loo-keng Hua in the west) personally served as chairman of the Beijing Competition Committee and engaged in the preparation of test materials. Many famous senior mathematicians, including Luogeng Hua, Zhongsun Fu, Jiangong Chen, Buqing Su, Xuefu Duan, and Zehan Jiang also made lectures during the competitions (Sun and Hu, 1994).

The development of Chinese mathematics competition activities can be divided into the following stages (cf. Chen and Zhang, 2013):

Stage I (1956–1964): Considered as the birth of China's competition mathematics, competitions were mainly advocated and personally directed by senior mathematicians and held in a few key cities of China.

Stage II (1978–1984): After the ten-year political turmoil came to an end, mathematics competitions were resumed. A working committee was set up by the Chinese Mathematical Society (CMS) in order to standardize and institutionalize the development of mathematics competition activities in China.

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Stage III (1985–present): This period sees a flourish of mathematics competition activities. China began to participate in the International Mathematical Olympiad (IMO) and in 1990, the 31st IMO was successfully held in Beijing on an unprecedented scale. The level of mathematics competitions in China quickly caught up with the international standard and continued to maintain its leading position afterwards. Meanwhile, various kinds of competitions at all levels were launched and a wide and diverse range of learning materials was readily accessible. The competition-oriented training became the "second classroom" for a proportion of students, or even the "second school" for a few top students.

1. Description of the Mathematics Competition Organization in China

Since the establishment of the Popularization Committee of the CMS and after more than three decades' explorations and practices, an ever-enriching, relatively stable system has evolved in the Chinese high school mathematical Olympiad practice. The National Senior High School Mathematics Competition and National Junior High School Mathematics Competition are two important events in this framework. Over the selection of the IMO national team, the CMS has also gradually set up a feasible working procedure.

1.1. National senior high school mathematics competition

In 1980, with a view to popularize mathematics on a national scale, the CMS held its first national conference in Dalian, in which organizing mathematics competitions was confirmed to be a regular task for the CMS and its branches in each province, municipality and autonomous region. In addition, the National Senior High School Mathematics Competition was agreed to be held in October annually (Zhu, 2009). Since 2014, the competition dates have been adjusted to be in September.

The National Senior High School Mathematics Competition is divided into two parts: Test 1 and Test 2 (also called the additional test). Test 1 mainly aims at educational spreading and popularization. Problems are mainly based on the *Mathematics Syllabus for Full-time Senior High Schools* (cf. Ministry of Education PRC, 2000), thus closely related to what students learn in class, but require some problem-solving strategies as well as a flexibility in knowledge applications. On the other hand, problems in Test 2 largely focus on the capability enhancement, some of which are specially designed for testing the mathematical ability and identifying mathematical talents. Contents are linked to the IMO, covering geometry, algebra, number theory and combinatorics. The overall difficulty of such test problems is lower than that of the IMO problems.

Before 2013, the set of test problems of the National Senior High School Mathematics Competition each year was carried out by the CMS in cooperation with a certain province. Since 2013, the CMS has been directly responsible for the test development. The quality of problems has been further improved.

Taking the 2013 National Senior High School Mathematics Competition as an example (cf. National Team Coaching Staff, 2014). Test 1 problems were largely related to the main knowledge topics like functions, inequalities, sequences and analytic geometry, which effectively covered each chapter in the textbooks and were designed at an appropriate difficulty tier. Test 2 focused on the extensible contents specified in the *Syllabus for Senior High School Mathematics Competition* (cf. Popularization Committee of the CMS, 2006a). The following four examples are taken respectively from Problems 1, 3 and 8 of Test 1 as well as Problem 3 of Test 2 of the 2013 National Senior High School Mathematics Competition.

Example 1.1. Let $A=\{2,\ 0,\ 1,\ 3\}$ and $B=\{x\,|\,-x\in A,\ 2-x^2\notin A\}$. Then the sum of all the elements in B is ______.

This problem only tests the basic knowledge of set representations. The first-year senior high school students may complete it successfully when they know the relevant concepts.

Since the elements in a set are unordered, $\{0,\ 1,\ 2,\ 3\}$ represents the same set as $\{2,\ 0,\ 1,\ 3\}$, but we are usually more used to the former representation because of its ordering from small to large. This problem intentionally disrupts the order of the elements. On the one hand, it is associated with the Year of 2013, which is lively. On the other hand, it is also an attempt to implicitly make the solver realize the unordered nature of set elements, and offer potential materials to the classroom teaching.

Example 1.2. Let ABC be a triangle with $\sin A=10\cdot\sin B\cdot\sin C$ and $\cos A=10\cdot\cos B\cdot\cos C$. Then the value of $\tan A$ is ______ .

The reference solving process of this problem is as follows:

Note that

$$\begin{aligned}\sin A-\cos A&=10(\sin B\cdot\sin C-\cos B\cdot\cos C)\\&=-10\cdot\cos(B+C)=10\cdot\cos A.\end{aligned}$$

Therefore, we have $\sin A=11\cos A$, which is equivalent to $\tan A=11$.

The problem is novel and beautiful with a natural statement. The formulae used to solve this problem are all basic contents of in-class instructions, and the reasoning chain is relatively short, which makes it a high-quality competition problem suitable for classroom teaching.

However, one cannot solve this problem by mechanically applying the formula in the textbook, nor by analyzing the two conditions in isolation. Instead, one should consider the conditions as a whole according to their structural characteristics, and find effective information combinations from them, so as to eliminate irrelevant quantities B and C. Therefore, this problem is also enlightening for the teaching of heuristics of mathematical problem solving.

Example 1.3. Let $a_1, a_2, \cdots, a_9$ be a sequence satisfying $a_1 = a_9 = 1$ and $\frac{a_{i+1}}{a_i} \in \left\{2, 1, -\frac{1}{2}\right\} (1 \le i \le 8)$. The number of sequences with such property is ______.

This is a counting problem designed with a sequence background, which is obviously much more difficult than exercises in textbooks, yet the knowledge and method involved are still within the teaching requirements of combinations and permutations, and the amount of calculation needed is appropriately controlled, so it would not be too challenging for students with clear thinking and good logic judgment.

Example 1.4. There are m problems in a test and n students taking the test, where m and n are integers greater than 1. For each problem, the scoring rule is as follows: If a total of x students fail to get a correct solution to this problem, those presenting correct answers will get x points, and the ones with wrong answers will receive zero points. The grade of each student is the total of the points gained from m problems. If the grades of all the students are written $p_1 \ge p_2 \ge \cdots \ge p_n$ from high to low, find the maximum possible value of $p_1 + p_n$.

Discrete mathematics is a relatively weak part in the current high school mathematics education in China, which leads to the incomplete display of students' mathematical talent. In competition mathematics, however, the materials of discrete mathematics are far richer. Especially, many of the combinatorics problems do not require specialized mathematical knowledge, but need imagination, insight and mathematical wit to some extent. Hence, discrete mathematics has an important value for the discovery of mathematically gifted students and the cultivation of their mathematical thinking.

Example 1.4 is a typical extremal value problem of discrete variables in competitions, but is unconventional for most students, since in-class instruction in senior high schools is mainly concerned with continuous variables. As a Test 2 problem, the knowledge and approach to be applied are both extended to some degree, but the problem is still mainly focused on mathematical thinking rather than specialized background knowledge. Generally, it requires the students to translate the condition into quantity, apply correctly the properties of inequalities and the mean value inequality of n unknowns to complete the upper bound estimation, as well as construct an optimal example.

The National Senior High School Mathematics Competition is a public-oriented extracurricular activity with significant influences. In recent years, there are approximately 50,000 students participating in this competition annually (around 1 million students in total if the preliminary competitions organized by each province or city are included). Moreover, the National Senior High School Mathematics Competition also has another function — to select best contestants around China (currently 350–400 annually) to participate in the China Mathematical Olympiad.

1.2. National junior high school mathematics competition

The National Junior High School Mathematics Competition is another public-oriented competitive event organized by the CMS to popularize mathematics. It aims to arouse students' interest in mathematical learning, develop their innovative awareness and capability, as well as discover and cultivate mathematical talents.

Since 1985, the National Junior High School Mathematics Competition has been held annually, normally in March or April (Zhu, 2009). The organization form is similar to that of the National Senior High School Mathematics Competition.

The contents of the National Junior High School Mathematics Competition cover numbers, algebraic expressions, equations and inequalities, functions, geometry, logic reasoning, etc., in which the contents listed in the mathematics curriculum standard are specified as the basic requirements of the competition (cf. Ministry of Education of the PRC, 2012). High requirements are made with regard to the comprehension level, flexibility in applications as well as the level of proficiency in the grasp of methods and skills. The competition similarly consists of Test 1 and Test 2. The former focuses on the basic knowledge and skills, while the latter focuses on the problem solving and analysis capabilities.

It is worth noting that though the contents of "Viete's theorem of quadratic equations," "criteria and properties of triangle similarity," "angle of circumference," "cyclic quadrilateral" and "tangent length of circle" have been diminished in the test requirements of the currently used curriculum standards, the mathematics competition for junior high schools still keeps these contents as a supplement for the in-class instruction. In the *Syllabus for Junior High School Mathematics Competition* (Popularization Committee of the CMS, 2006b), contents of "four concyclic points" and "circle power theorem" were added in particular besides the above-mentioned knowledge. Take Problems 1 and 2 in Test 2 (Paper A) of the 2014 National Junior High School Mathematics Competition as examples (cf. Xu, 2015):

Example 1.5. Let a, b be real numbers such that $a^2(b^2+1)+b(b+2a)=40$, and $a(b+1)+b=8$. Find the value of $\frac{1}{a^2}+\frac{1}{b^2}$.

Example 1.6. As shown in Fig. 1, in the parallelogram $ABCD$, point E is on diagonal BD such that $\angle ECD = \angle ACB$. The extended line of AC meets the circumcircle of triangle ABD at F. Prove that $\angle DFE = \angle AFB$.

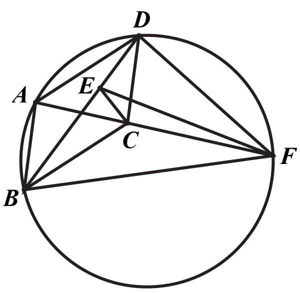

Fig. 1. A geometric problem in 2014 National Junior High School Mathematics Competition

In Example 1.5, the problem can be simplified by changing the variables $a+b=x,\ ab=y$. It is not difficult to see that the structural feature of algebraic expressions in this problem is closely related to "Viete's theorem of quadratic equations." In Example 1.6, the knowledge of "criteria and properties of triangle similarity" and "four concyclic points" is tested.

If the curriculum standards are meant for the fundamental goal of compulsory education, then competition mathematics can be reckoned as an extension, in both breadth and depth, for students who have the capacity and desire for further study, to develop their thinking ability and avoid a large amount of low-level repetition in mathematical learning. The training in algebraic techniques can improve their ability in algebraic calculations; the knowledge of circles and similar triangles can provide abundant mathematical materials for the in-class instruction and after-class study, and further improve students' ability of geometrical reasoning. It should also be noted that, these are directly related to analytic geometry and other contents to be studied in senior high school.

1.3. Chinese Girl's Mathematical Olympiad

In many mathematics competitions, there are always more men and fewer women among the competitors. Traditionally, many people think that boys are generally better than girls in mathematics. Although this statement lacks the support of the actual research data, the fact that the numbers of boys and girls of the mathematical Olympians are out of balance promotes the birth of the "Chinese Girl's Mathematical Olympiad."

In August 2002, China Mathematics Olympic Committee of the CMS held the first Chinese Girl's Mathematical Olympiad in Zhuhai. The participants were girls in senior high schools. The purpose of this activity was to show their mathematical talent and other talents, set up a stage for them, increase their interest in learning mathematics, improve their mathematics learning level, and promote their mutual learning and friendship in different regions.

Academician Wang Yuan, a famous mathematician, inscribed the Chinese Girl's Mathematical Olympiad: "Sophie Germain, Sofya Kovalevskaya, and Emmy Noether, the names of these great women mathematicians and their outstanding achievements are enough to prove that women have very high mathematical talent, which is certainly suitable for studying mathematics."

It has been 18 years since the first Chinese Girl's Mathematical Olympiad was held in 2002. Through the activities of the 18th Session, it provides a platform for many excellent high school girls to appreciate, study and research into mathematics, and also trains a large number of outstanding girls.

The Chinese Girl' s Mathematical Olympiad is held once a year and has been held for 18 times. The competition time is in the middle of August every year. There are about 40 teams participating in each competition, each team has 4 participants. The United States, Russia, the Philippines, Singapore, the United Kingdom, Japan, South Korea, and Hong Kong, Macao and Taiwan of China have also sent teams to participate in the Chinese Girl's Mathematical Olympiad. The Chinese Girl's Mathematical Olympiad has a written test for two days which is in line with the IMO. Four problems need to be solved within 4 hours each day.

The scope of test problems for the Chinese Girl's Mathematical Olympiad is the same as the IMO, involving algebra, geometry, combinatorics and number theory, but the difficulty is lower than the IMO. The competition awards the first place in the total score of the group and the gold, silver and bronze medals of the individuals. From 2012, the top 12 students in total score are directly invited to participate in the Chinese Mathematics Olympiad.

In order to enrich the life of the contestants, cultivate their creativity and the team spirit, the Chinese Girl's Mathematical Olympiad has specially designed the girls' aerobics competition, which was planned when we held the first Chinese Girl's Mathematical Olympiad in Zhuhai, and has persisted to today.

1.4. China Western Mathematical Invitation

The China Western Mathematical Invitation began in 2001, shortly after the country launched the western development, with the original name "China Western Mathematical Olympiad."

At the beginning of China's participation in the IMO, there were participating students from Western China almost every year. In 1989, the national team even had four participants from the West. But since 1991, the western region was quite silent for ten years. At the same time, the performance in both the National Senior High School Mathematics Competition and the China Mathematical Olympiad showed obvious gap between Western China and Eastern China.

In order to "maximize the mobilization of schools in the western region to actively participate in the mathematics competition" and promote the improvement of the level of mathematics and science education in the western region, Zonghu Qiu, who has been doing "what I am willing to do," put forward the idea of holding the mathematics competition in the western region.

The first China Western Mathematical Olympiad was held in the ancient city of Xi'an in early November 2001. The teams of high school students from 11 provinces, cities and districts in the West, and Shanxi, Jiangxi, Hainan and Hong Kong participated in the competition. The participants of the China Western Mathematical Olympiad are mainly grade 11 and grade 10 students. The competition is divided into two days. Four problems need to be solved within 4 hours each day. The overall difficulty is roughly equivalent to that of the additional test of the National Senior High School Mathematics Competition. Before 2012, the first and second place winners in each competition could directly participate in the training of the national training team.

Since 2012, the China Western Mathematical Olympiad has been renamed the China Western Mathematical Invitation. The competition dates have been changed from the first half of October to the middle of August since 2013.

The development of this competition reignited the enthusiasm of Western students for mathematics. Once again, the figure of Western students often appears in the national team.

This competition has also attracted the representatives of Singapore, Indonesia, Malaysia, Philippines and other countries to take part in successively. Kazakhstan, as a strong team in IMO, has participated in each of the following competitions since 2003, and has often sent IMO gold and silver medalists to participate in this competition.

1.5. Selection procedures for the IMO Chinese national team

The following figure (Fig. 2) illustrates the selection procedure for the IMO Chinese national team.

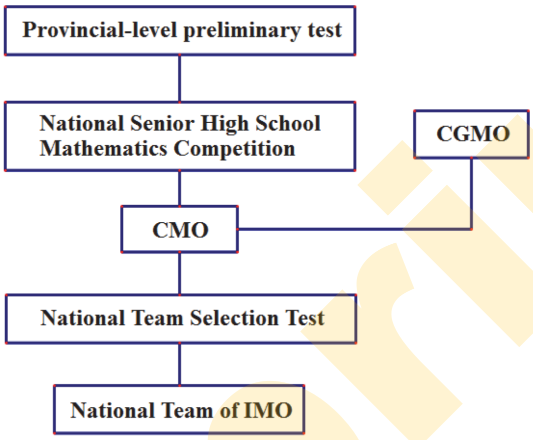

Fig. 2. The selection procedure for the IMO Chinese national team

In the figure, CMO refers to "China Mathematical Olympiad," a mathematical competition organized by the Mathematical Olympic Committee of the CMS aiming to select mathematical talents. It is also the top mathematics competition for high school students in China. CGMO refers to the "China Girl's Mathematical Olympiad."

The participants in the CMO consist of winners of the National Senior High School Mathematics Competition from each province, the CGMO winners and the invited international teams. The top 60 participants from mainland China have qualifications to enter the National Training Team and they also qualify for admissions into any top university in China. The difficulty of the CMO test is roughly the same as the IMO.

As the final round of the entire selection process, the National Team Selection Test aims to select 6 members for the national team of the IMO. Usually this round consists of several sets of extremely difficult problems.

1.6. Performance of the IMO Chinese National Team

China took part in the 26th IMO in 1985 for the first time, when only two students went there. Since 1986, with the exception of the one held in Taiwan in 1998, the Chinese team has sent six students to participate in the IMO. The following Fig. 3 and Tab. 1 are about China's participation in the IMO.

Fig. 3. Team results of the IMO Chinese national team

Tab.1. Individual results of the IMO Chinese national team

Year	Team Size			P1	P2	P3	P4	P5	P6	Total	Rank		Awards				Leader	Deputy leader
	All	M	F								Abs.	Rel.	G	S	B	HM		
2019	6	6		40	41	27	41	42	36	227	1	100.00%	6	0	0	0	Bin Xiong	Yijie He
2018	6	6		42	37	17	42	42	19	199	3	98.11%	4	2	0	0	Zhenhua Qu	Yijie He
2017	6	6		42	25	0	42	19	31	159	2	99.09%	5	1	0	0	Yijun Yao	Sihui Zhang
2016	6	6		42	30	20	42	42	28	204	3	98.15%	4	2	0	0	Bin Xiong	Qiusheng Li
2015	6	6		42	36	12	42	22	27	181	2	99.03%	4	2	0	0	Bin Xiong	Qiusheng Li
2014	6	6		42	42	16	42	35	24	201	1	100.00%	5	1	0	0	Yijun Yao	Qiusheng Li
2013	6	6		42	38	30	41	42	15	208	1	100.00%	5	1	0	0	Bin Xiong	Qiusheng Li
2012	6	6		42	40	14	31	38	30	195	2	98.99%	5	0	1	0	Bin Xiong	Zhigang Feng
2011	6	6		42	12	42	42	42	9	189	1	100.00%	6	0	0	0	Bin Xiong	Zhigang Feng
2010	6	5	1	41	42	23	42	24	25	197	1	100.00%	6	0	0	0	Bin Xiong	Zhigang Feng
2009	6	6		42	42	42	42	42	11	221	1	100.00%	6	0	0	0	Huawei Zhu	Gangsong Leng
2008	6	5	1	42	42	42	42	35	14	217	1	100.00%	5	1	0	0	Bin Xiong	Zhigang Feng
2007	6	6		36	42	17	41	42	3	181	2	98.91%	4	2	0	0	Gangsong Leng	Huawei Zhu
2006	6	6		42	42	35	41	38	16	214	1	100.00%	6	0	0	0	Shenghong Li	Gangsong Leng
2005	6	6		32	42	35	42	42	42	235	1	100.00%	5	1	0	0	Bin Xiong	Jianwei Wang
2004	6	6		37	41	21	42	37	42	220	1	100.00%	6	0	0	0	Yonggao Chen	Bin Xiong
2003	6	6		42	40	28	42	42	17	211	2	98.77%	5	1	0	0	Shenghong Li	Zhigang Feng
2002	6	6		41	41	24	42	42	22	212	1	100.00%	6	0	0	0	Yonggao Chen	Shenghong Li
2001	6	6		42	40	23	42	42	36	225	1	100.00%	6	0	0	0	Yonggao Chen	Shenghong Li
2000	6	6		42	42	19	39	42	34	218	1	100.00%	6	0	0	0	Jie Wang	Yonggao Chen
1999	6	6		40	41	11	42	33	15	182	1	100.00%	4	2	0	0	Jie Wang	Jianping Wu
1997	6	6		39	42	38	38	41	25	223	1	100.00%	6	0	0	0	Jie Wang	Jianping Wu
1996	6	5	1	34	42	32	36	0	16	160	6	93.24%	3	2	1	0	Wuchang Shu	Chuanli Chen
1995	6	5	1	42	42	40	36	42	34	236	1	100.00%	4	2	0	0	Zhusheng Zhang	Jie Wang
1994	6	6		42	42	42	42	40	21	229	2	98.53%	3	3	0	0	Xuanguo Huang	Xingguo Xia
1993	6	6		35	42	34	39	39	26	215	1	100.00%	6	0	0	0	Lu Yang	Xilu Du
1992	6	6		41	42	42	38	35	42	240	1	100.00%	6	0	0	0	Chun Su	Zhenjun Yan
1991	6	6		42	42	31	42	42	32	231	2	98.18%	4	2	0	0	Yumin Huang	Hongkun Liu
1990	6	6		42	42	35	42	40	29	230	1	100.00%	5	1	0	0	Zun Shan	Hongkun Liu
1989	6	5	1	40	42	37	41	42	35	237	1	100.00%	4	2	0	0	Xiwen Ma	Zun Shan
1988	6	5	1	42	41	17	42	42	17	201	2	97.92%	2	4	0	0	Gengzhe Chang	Wuchang Shu
1987	6	5	1	31	42	27	38	40	22	200	8	82.93%	2	2	2	0	Xiangming Mei	Zonghu Qiu
1986	6	5	1	30	42	14	29	30	32	177	4	91.67%	3	1	1	0	Shouren Wang	Zonghu Qiu
1985	2	2		8	14	0	4	0	0	26	32	16.22%	0	0	1	0	Shouren Wang	Zonghu Qiu

Many outstanding young mathematics talents have emerged in China's mathematics competitors, such as Wei Zhang, Zhiwei Yun, Chenyang Xu, Yifeng Liu, etc. who have won the famous Ramanujan prize. Many scholars, such as Xinwen Zhu, Song Wang, Ruochuan Liu, Hongyu He, Simai He, Xinyi Yuan, Liang Xiao, etc., have been engaged in mathematical teaching and research at well-known universities or scientific research institutions in China and abroad, and made a great professional job. Wei Dongyi, who won the full mark gold medals of the IMO in 2008 and 2009, made good achievements in the first and second year of his postgraduate studies.

In China, 6 people won the Paul Erdös award, including Zonghu Qiu (1994), Wen-Hsien Sun (2002), Shian Leou (2008), Kar-ping Shum (2016), Bin Xiong (2018) and Gangsong Leng (2020).

2. Training Systems of Mathematics Competitions in China

Along with various mathematics competitions, extensive educational activities related to these competitions have been organized all over China and a comprehensive training system has taken shape gradually. As a result, China's performances in mathematics competitions have improved a lot and have gradually formed international influence.

2.1. Organizational forms of school-level training

School-level training is the foundation for Chinese education on competition mathematics. It mainly adopts a form of "second classroom," such as interest groups and extension courses etc., which is accessible by each grade of high school. Such training activities mainly serve as a means of popularization for mathematics competitions and a supplement for the classroom study. However, at some key schools, school-level training is highly specialized and these schools have become home bases for training high-level Mathematical Olympiad participants.

It is generally accepted that gifted education mainly consists of three basic forms: enrichment, differentiation and acceleration (Sheng and Zhou, 2010). Enrichment refers to enhanced study materials for gifted students, so as to extend their scope of knowledge. Differentiation refers to grouping students according to their capabilities with adapted courses to suit the needs of students at different levels. Acceleration means intensive materials or more information for students, so as to accelerate the pace of teaching, and let gifted students acquire the advanced knowledge system as soon as possible. The three forms of training coexist in the school-level training for mathematics competitions.

For enrichment, to be specific, schools offer mathematics extension courses with different themes that include competition mathematics, mathematical problem solving, mathematical modeling and application, history and culture of mathematics. Schools offer a broad variety of choices to students with different interests and at different levels. These are beneficial to enhancing students' mathematical understanding, improving their mathematical literacy and broadening their horizons. Moreover, some schools also organize competition mathematics groups among gifted students by conducting extension lectures or training after normal school hours. Participation in this kind of activities is voluntary and mainly for mathematics competitions.

For differentiation, at some key schools, students with a mathematical talent are assigned to a certain class to receive separate small-size class teaching. With a tradition of specializing in competition mathematics, schools provide accelerated mathematics courses in each grade. The overall teaching plan for the entire semester is mapped out at the time of admissions, and further differentiated teaching is adopted for students in the above accelerated programs. Shanghai High School, for example, has adopted differentiated teaching since 1990 and successfully trained the IMO gold medalists for consecutive years starting from 2008. Every year, the school selects around 40 freshmen who are mathematically talented to form an experimental class, and designs specific courses for them. Since 1998, it began to further select about 10 most gifted students from the experimental class to teach them in small class. Finally, individual instruction is offered to 3–4 exceptionally gifted students who have shown great potentials (Tang and Feng, 2011).

In many instances, differentiation is to accelerate the teaching process for gifted students, i.e., to let students quickly master textbook knowledge as well as the basic knowledge and skills required in mathematics competitions, so as to lay a better foundation. After this is completed, students are at liberty to further study for competition mathematics and improve their skills in solving mathematical problems.

Among the above three forms of gifted education on mathematics, enrichment mainly focuses on popularization of competition mathematics, while differentiation and acceleration serve the functions of both popularization and improvement. The latter two forms do not apply to most students so as to avoid excessive academic burdens. However, they are suitable for students who can easily master mathematical knowledge and skills without adversely affecting the study of other subjects.

For high school mathematics competitions, many schools such as Shanghai High School adopt a tutor teaching model of "$1+n$" (Tang and Feng, 2011), which is to assign a long-term mathematics teacher for a certain group of gifted students. This model consolidates the collective wisdom of the school's mathematics tutoring team and sometimes even a guest expert team. The core teacher must have intimate knowledge of every student to assess their potentials, follow closely their performances in school and formulate individualized instructional plans. He or she must also set up a specific timetable for each milestone and invite professional experts to offer special guidance at appropriate times. This model has several advantages, namely, putting equal emphasis on students' foundation and improvement, showing students how different teachers have different perspectives and thinking styles of mathematical problem solving.

School-level training normally starts from classroom teaching to the popularization and promotion of competition mathematics. This whole process can be roughly divided into four stages, i.e., basic training, thematic training, intensified training and pre-competition training (Feng, 2006). The emphasis for each stage is different.

The basic training stage aims to finish teaching the contents of high school mathematics, thus ensuring that students reach the level required for graduation. Forms and methods of teaching are similar to conventional ones, but contents are intensified and expanded to some degree, with an emphasis on basic thinking skills used in competition problems. Training at this stage enables students with a certain mathematical ability to perform well in public-oriented competitions.

The main task of the "thematic training" stage is to systematically impart knowledge and problem solving skills required in mathematics competitions. Teachers need to encourage students to form good study habits, further improve their self-learning awareness and ability, as well as keep a positive learning attitude. After this stage, students can gain a deeper understanding of various topics in competition mathematics, be able to solve difficult problems such as those in the additional test of the National Senior High School Mathematics Competition.

Intensified training is generally targeted at a few outstanding students (mainly the CMO and higher-level participants). At this stage, challenging problems can be used for exercises, to keep top students moving forward by competing and cooperating with each other.

Pre-competition training is usually aimed at preparing students for a certain competition. Well-prepared simulation tests not only enable students to familiarize themselves with forms of competition, but also reveal students' lack of knowledge and skills so as to provide reference for them and their teachers. A week or two prior to some provincial or higher level mathematics competitions, a lot of schools will organize intensive training for participants during the vacation time and after-class hours. This is also an integral part of school-level training.

Generally speaking, the first stage is closely related to the usual curriculum, while the second and the third mainly put emphasis on skill improvements, and the fourth mainly focus on competitive readiness.

2.2. Provincial and municipal-level competitions and training organizations

There are many high school mathematics competitions at provincial and municipal levels in China. Such competitions are usually hosted by mathematical societies or associations in relevant provinces and cities. In Beijing and Shanghai, these competitions have become traditional events. In some provinces, the provincial mathematics competition is reckoned as preliminary tests for the National Mathematics Competition.

In addition to hosting competitions, provincial and municipal mathematical societies also organize training programs for mathematics competitions.

Many provinces organize mathematics summer camps, and invite university professors, senior teachers and trainers to teach participating students. In Zhejiang province, for example, the Department of Mathematics of Zhejiang University and Zhejiang Mathematical Society jointly hold mathematics summer camps every year, and organize training courses at levels from college entrance exams to the National Senior High School Mathematics Competition, in order to improve students' performances in competitions and university entrance exams.

The various provincial and municipal mathematics summer camps are popular among high school mathematics enthusiasts, and often attract students from other provinces and cities to participate. These camps have turned out thousands of talented mathematics students, among which there were IMO and CMO winners as well as top candidates in the National College Entrance Examinations.

A distinctive model can be found in Shanghai — the Secondary Extracurricular Mathematics School. Thanks to the training of the Secondary Extracurricular Mathematics School, students in Shanghai have got great achievements in domestic and international mathematics competitions in the recent two decades and more. The school is hosted by the Shanghai Mathematical Society and guided by the Teaching and Research Office of the Shanghai Municipal Education Commission. Since its establishment in 1987, the school has been run for more than twenty years, and the late mathematician Chaohao Gu used to be the honorary principal of this school. Students come from the sixth to twelfth grades, with approximately 400 students from each grade, all of whom are Shanghai's students talented in mathematics. The school also invites professors from Fudan University, East China Normal University and the best local trainers for mathematics competitions. Every Sunday, the school offers 2 hours of extracurricular instruction on mathematics.

Courses offered by this school are roughly synchronized with general courses by high schools, conforming to the "enrichment" form of gifted education. Here, students not only have the opportunity to strengthen what they have already learnt in the classroom, but also to learn new knowledge, such as knowledge in elementary number theory and combinatorics, some famous theorems in Euclidean geometry, as well as mathematical problem-solving strategies and skills. Most students can quickly master them after teachers' instructions and group discussions.

Furthermore, entrusted by the Shanghai Mathematical Society, the school has also been responsible for the proposition and organization of competitions and selected some exceptionally gifted students to receive special tutoring. Since 1987, almost all national team members coming from Shanghai have experiences of training in this school.

With a combination of provincial, municipal and school-level training, both mathematically gifted students and their tutoring team are offered the opportunity for further development.

2.3. *The training system for mathematics competitions*

Generally, the structure of training systems for China's mathematics competitions is of "school level — provincial and municipal level — inter-provincial level" as shown in Fig. 4, balancing popularization and improvement, and covering students of different ages and learning levels. Inter-provincial mathematics competition activities consist of regional mathematics competitions and a variety of summer camps, which can generate many shared resources each year.

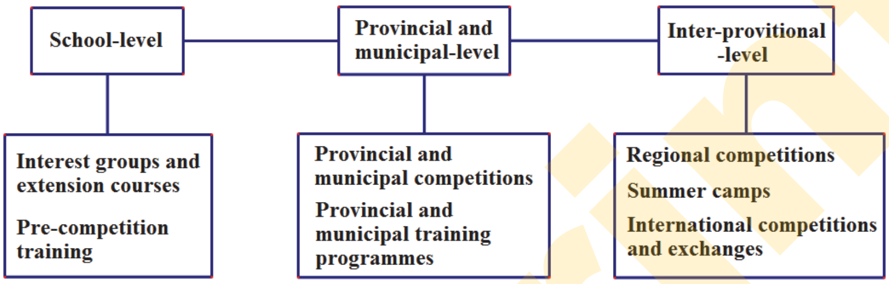

Fig. 4. The training system for China's mathematics competitions

Moreover, the training of tutors is also a part of the overall training system. The Popularization Committee of the CMS established a "China Mathematical Olympiad Tutor Rating System" in 1988. The CMS and its branches in each province organize training workshops for high school teachers who have few experiences in teaching competition mathematics, then evaluate their performances through their problem-solving skills and teaching plans. Tutors will receive some extra learning opportunities before workshops in addition to basic courses and teaching internship on mathematics education. Some normal universities also offer optional courses such as competition mathematics, problem-solving strategies and mathematical methodology to them for future developments.

2.4. *Extensions of mathematics competition training*

As a way of mathematics gifted education, the mathematics competition training aims at fostering future mathematicians and scientific elites in the long run. Many mathematics educators and practitioners pay attention to encouraging high school students and teachers to conduct research-oriented learning and academic communications in mathematics competition activities.

In this regard, the New Star of Mathematics (NSMATH) is an influential training project in recent years. It was founded in January 2014 by Professor Leng Gangsong, one of the winners of the 2020 Paul Erdös award. Its current official website is http://www.nsmath.cn/.

The website of NSMATH has several columns, among which the most distinctive one is "Problems for Solutions." The problem proposers are mainly high school students, coaches and young mathematicians, many of whom are former contestants, even IMO gold medalists. The selected problems are very challenging and have profound mathematical meaning, with training and research values for the students who take part in the high-level mathematics competitions at home and abroad. From the 13th issue, this column has been in the charge of Mou Xiaosheng (2008 IMO gold medalist, Ph.D. from Harvard University). The novelty and difficulty of the selected problems have been further improved, making this column more acclaimed.

"Students' Works" is another brilliant column of this website. Students contribute enthusiastically with articles full of new ideas and methods, reflecting the strong creativity of high school students. Many of their articles are concerned, discussed and carefully modified by experts and scholars. In this way students' research interests are greatly stimulated and their research ability is also improved. Nowadays students are proud to be able to publish in this column.

By the end of 2019, NSMATH has published 35 issues of "Problems for Solutions" and more than 200 articles. The website has grown into a high-quality mathematics competition network, which not only reflects the mathematics innovation ability of high school students, but also encourages students and teachers to conduct research-oriented learning in mathematics competition activities. NSMATH has also held many mathematics competitions learning camps. The learning camps have first-class tutor resources and a large number of excellent students, which ensures a high level of these activities and provides valuable opportunities for the excellent students and teachers around the country to face up to each other.

The programs such as NSMATH are of long-term significance to the development of high school mathematically gifted students in China, as well as to the improvement of coaches' ability and career development.

3. Developing Ability through Competition Mathematics

Mathematical ability refers to a relatively stable psychological characteristic for the successful completion of mathematics activities. Mathematicians, educators and psychologists at home and abroad have discussed mathematical ability from different aspects.

Krutetskii (1976) has determined the 9 key elements of mathematical ability according to the basic features of mathematical thinking:

1. The ability to formalize mathematical materials, and operate in the formal structure.
2. The ability to summarize mathematical materials.
3. The ability to operate by using numbers and other symbols.
4. The ability to use continuous and rhythmic logical reasoning.
5. The ability to shorten reasoning process.
6. The ability to reverse psychology process (ability of transferring from positive thinking to reverse thinking).

7. Flexibility in thinking.
8. Mathematical memory.
9. Spatial concept.

In the Principles and Standards for School Mathematics released by the American National Council of Teachers of Mathematics (NCTM, 2000), "problem-solving, reasoning and proof, communication, connection and representation" are defined as the five criteria of process ability in mathematics.

In 2003, it was pointed out by the Mathematics Curriculum Standard for Senior High Schools (Experimental) that when people were learning mathematics and using mathematics to solve problems, they constantly underwent thinking process such as intuitive perception, observation and discovery, analogical induction, spatial visualization, abstraction, symbolic representation, operation and problem solving, data processing, deductive reasoning, reflection and construction (Ministry of Education of the PRC, 2003). It also indicated that mathematical thinking played a unique role in the formation of rational thinking.

Competition mathematics education is beneficial to the development of gifted students' mathematical ability in various aspects.

3.1. Mathematical ability and problem solving

We may as well combine with the NCTM standards to give some explanations to the educational value of competition mathematics.

In recent years, the teaching of geometry has been weakened in the compulsory education in China. The lack of training in deductive reasoning hinders the development of students' reasoning skills, which arouses the concern of mathematicians and mathematics educators. Yet in mathematics competitions at all levels, the proportion of geometry problems remains stable, especially in the IMO of recent years, where team leaders from all countries often select some extremely difficult geometric problems as official competition problems. It is somewhat beneficial to maintain the level of reasoning and proving of those mathematically gifted.

In addition, mathematics competition problems require students to recognize and use "connections" with flexible mathematical thinking at a more advanced level. At the same time, they require students to be good at selecting, applying and converting mathematical representations, which contributes to the enhancement of students' ability in mathematical connections and mathematical representations.

Furthermore, mathematics competition problems require students to present their mathematical ideas and problem-solving processes clearly. Students should also evaluate others' mathematical thinking and problem-solving processes by communicating with teachers and classmates. These activities are beneficial for improving students' ability in mathematical communications.

More importantly, competition mathematics provides rich sources for mathematical problem solving. It is pointed out by Luogeng Hua that "the nature of mathematics competition is different from that of an exam in school, and also not the same as the university entrance exam. What we require is that students taking part in the competition can not only apply formulae and theorems, but also show their flexibility in thinking, a good understanding of mathematical principles, and the ability to use these principles to solve problems. They should even be able to discover new methods and principles to solve unfamiliar problems. Such a requirement can exactly test and train the students' mathematical ability" (Hua, 1956b, p. 1).

Example 3.1. Ten numbers $1, 2, \cdots, 10$ are written by order on the circumference initially (10 adjacent to 1). Two types of operations are permitted: (a) swap the locations of two adjacent numbers, (b) allow any two adjacent numbers to plus an integer at the same time. Is it possible to turn all numbers into 10 by finite steps of such operations?

This is a relatively easy competition problem. The authors of this article once put forward this problem to senior high school students who had not undergone competition mathematics training. The students made repeated attempts and found that it was always so close to the "goal" but failed. They hardly caught the key point of the problem. Some of them were confident that the conclusion was "impossible," but they struggled to carry out the mathematical reasoning. As a matter of fact, one might consider the invariance on the whole: No matter operating through (a) or (b), the sum of the ten numbers always maintains the original parity. Teachers often guide students in problem-solving strategy, inspire them to be aware of all-rounded considerations and to look for the quantitative relationship which remains unchanged in the operation.

Example 3.2. Given real numbers m, x, y, with $x, y > 0$ and $x + y < \pi$. Prove that

$$(m^2 - m)\sin(x+y) + m(\sin x - \sin y) + \sin y > 0.$$

This problem is somewhat difficult. The challenge lies in the complex structure (a quadratic form mixed with a trigonometric form), too many parameters (three in total), and not knowing how to use the condition $x, y > 0$ and $x + y < \pi$. Here is an illuminating solution:

Step 1. Construct a triangle ABC with $A = x$, $B = y$, $C = \pi - x - y$. Let $a = BC$, $b = CA$, $c = AB$.

Step 2. Applying the law of sines, one has

$$\frac{\sin x}{a} = \frac{\sin y}{b} = \frac{\sin(\pi - x - y)}{c} = \frac{\sin(x+y)}{c} > 0.$$

Therefore, the initial inequality can be transformed into

$$(m^2 - m)c + m(a - b) + b > 0.$$

Step 3: Since $c > 0$, it suffices to prove that the discriminant of the quadratic form $cm^2 + (a - b - c)m + b$ with respect to m is negative, which is equivalent to $(a - b - c)^2 - 4bc < 0$, or

$$a(a - b - c) + b(b - c - a) + c(c - a - b) < 0,$$

which is an easy consequence of $a < b + c,\ b < c + a,\ c < a + b$.

Throughout the above solution, the original problem has been translated and reduced step by step. The first step is an application of the idea of "construction," which allows the subsequent steps to be removed from a lot of complex calculation. The second step is due to the conditions $x, y > 0,\ x + y < \pi$ and the homogeneity of the law of sines. The third step is due to the quadratic structure of the inequality with respect to m.

When explaining problems such as Example 3.2, teachers should guide students to grasp the structure of the algebraic expression, to make associations and connections with familiar knowledge in order to simplify the original problem. A helpful tip is to apply the idea of "construction," which often enables the problem-solving process to be "simple and delicate."

Solving problems by a constructive method requires comprehensive knowledge, divergent thinking and keen intuition. The following Example 3.3, a problem quite impressive in the authors' teaching experience, is also related to the constructive method.

Example 3.3. For any given positive integer m, prove that there exist $2m+1$ positive integers a_i $(1 \le i \le 2m+1)$ making up an increasing arithmetic progression, so that the product of these integers is a perfect square (Xiong and He, 2012).

The solution can be done in a sentence: Let $a_i = ik$ $(1 \le i \le 2m+1)$, where $k = (2m+1)\,!$, so that $a_1 a_2 \cdots a_{2m+1} = (2m+1)!\,k^{2m+1} = (k^{m+1})^2$.

However, the brief answer above does not reflect the hidden thinking process. How can this problem be considered? In fact, one can start from an arbitrary increasing arithmetic progression with $2m+1$ positive integer terms $b_1, b_2, \cdots, b_{2m+1}$.. Note that for any positive integer k, $b_i k$ $(1 \le i \le 2m+1)$ also make up an increasing arithmetic progression. Therefore, one can freely select the value of k to satisfy the condition "product is a perfect square". For instance, one can let $k = b_1 b_2 \cdots b_{2m+1}$, and then $(kb_1)(kb_2)\cdots(kb_{2m+1}) = (k^{m+1})^2$.

In brief, we have adopted a frequently used strategy that "relax a condition and then try to re-impose it," which is typically helpful in solving a number of construction problems.

The problem is not as easy as it seems in solution. Once we set this problem to a number of senior high school students with some experience in mathematics competition. We observed that most students considered this issue from the following two perspectives: The first perspective was starting from the simple cases (for example, the case of 3 terms or 5 terms, and then tried to make a generalization). However, it would seem difficult for these students to extract the general rules. The second perspective was to set out the two basic parameters of arithmetic progression (such as the common difference d and the middle term $a = a_{m+1}$), expressed the product of $2m+1$ terms, and then tried to solve the equation with three unknowns. We noticed that some students gazed longingly at the equation and got stuck because of the complexity of the structure. Only a few of them completed the construction within 20 minutes.

Indeed, in solving such a problem, a great deal of thinking is needed. Students should not only strive to plan their solving processes, but should always monitor their thinking to determine the feasibility of the scheme, and avoid the interference of invalid plans. Such kind of experience is beneficial for students to improve both their problem solving skills and their metacognitive monitoring strategies.

For ease of comparison with Example 3.2 and Example 3.3, the following is a list of three problems relating to the same knowledge point:

Example 3.4. Determine all possible real numbers k such that $kx^2-(k-2)x+k>0$ holds for any real number x.

Example 3.5. In triangle ABC, the circumcircle radius is $\sqrt{2}$, and

$$2\sqrt{2}\left(\sin^2 A-\sin^2 C\right)=(a-b)\sin B.$$

Find the degree of angle C and the maximum value of the area of triangle ABC.

Example 3.6. Let $\{a_n\}$ be an infinite arithmetic progression. For any positive integer n,, denote the sum of $a_1, a_2, \cdots, a_n$ by S_n.

(1) If $a_1=\dfrac{3}{2}$ and the common difference of $\{a_n\}$ is 1, find the positive integer k that meets $S_{k^2}=(S_k)^2$.

(2) Find all $\{a_n\}$ such that $S_{k^2}=(S_k)^2$ for every positive integer k.

The three problems above are selected from the book Mathematics Review Guide for College Entrance Examination (Office of High Schools of the REP, 2006). They roughly correspond to the difficult problems in classroom teaching. The knowledge and method involved in Example 3.4 and Example 3.5 are close to that of Example 3.2, but the direction in problem solving is somewhat clear. In Example 3.6, one can apply a routine method to find the answer of problem (1). The corresponding groups (a_1, d)

of problem (2) can be inferred from $k = 1, 2$ before a complete verification, or one can also directly write the quadratic equation of k, and solve it by the principle of polynomial identity. Both solutions require strong computational and reasoning skills, however, a specific formula and method can be applied at each step. In contrast, although no more knowledge is needed in Examples 3.2 and 3.3, the two problems are not conventional in classroom teaching and require much more thinking. Moreover, Example 3.1 only relates to the knowledge of addition and subtraction, however, it is still good material for high school students to enrich their thinking patterns.

For those who have a certain understanding of competition mathematics, most feel that even if one is able to read the methods and principles for 100 problems, one could still feel helpless when facing the 101st problem. In a sense, many mathematics competition problems are special or even unique, thus simple imitation does not always work. In fact, students should be able to transfer the thinking method and problem-solving strategy in one case to other cases by imitating and practicing, in which way they will not be at a loss when solving new problems in mathematics or even other disciplines. At the same time, they should also be able to generate new ideas and try different methods, which is in line with the basic spirit of problem solving.

3.2. Competition mathematics and teaching of open-ended mathematics problems

Teaching of open-ended problems is another major issue related to problem-solving and creativity. It was first introduced from Japan by Zaiping Dai who was a strong advocate of it. Since then, abundant theoretical achievements have been obtained in China (cf. Dai, 2002).

Roughly speaking, open-ended mathematics problems are those with non-exclusive answers. In most cases, there is only one correct answer to a high school mathematics problem; even for "a problem with several solutions," there are only a few which would be easy to arrive at. However, it seems that problems posed in higher-level mathematics competitions are more likely to be open-ended in terms of the problem-solving approach, with solutions often beyond the problem proposers' anticipation.

For example, among the four problems in the additional test of the 2013 National Senior High School Mathematics Competition, the solutions of three problems were simplified afterwards. In the 2014 National Team Selection Test, the examining committee noted some brilliant solutions after going over the answer sheets, and the given reference answers to several other problems were also simplified, which accounted for more than half of the total. In the Southeast China Mathematical Olympiad in 2014, a much simpler solution, unexpected by all experts from the examining committee, was discovered by a few of participants to unlock the most difficult problem. Historically, in the 1980 joint competition of Finland, the United Kingdom, Hungary and Sweden, the solution of a certain problem was rather complicated where mathematical induction was used four times, the Chinese translation of which took around 4,000 characters. Later Chinese experts gave some simpler solutions, the lengths of which were just a dozen lines (cf. Zhu, 2009). However, the relevant algebraic skills were deeply concealed and difficult to be discovered.

In all previous sessions of the IMO, the examining committee would present a special award to those participants who gave particularly brilliant solutions and non-trivial generalizations. To date, this special prize has been awarded to more than 40 participants.

Below is a case from the authors' personal teaching experience:

Example 3.7. Let ABC be a triangle with area S and circumcircle radius R.

(1) Prove that $S = 2R^2 \sin A \sin B \sin C$..

(2) Prove that $S = \dfrac{R^2}{2}(\sin 2A + \sin 2B + \sin 2C)$.

This problem was designed for a test just after the completion of teaching, which was intended to test students' flexible application of the formula of triangle's area, the law of sines and the sum-to-product formula. The anticipated method was to use the formula of triangle's area $S = \dfrac{1}{2}ab\sin C$ (or $S = \dfrac{abc}{4R}$) and $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ $= 2R$ (the law of sines) to finish problem (1), and further to solve problem (2) by deducting the identity $\sin 2A + \sin 2B + \sin 2C = 4\sin A \sin B \sin C$ in the triangle.

One of the students came up with a different approach to solve problem (2):

Denote the circumcenter of triangle ABC by O. The student first assumed that ABC was an acute triangle. In this case,

$$S_{\Delta AOB} = \frac{1}{2} OA \cdot OB \cdot \sin\angle AOB = \frac{1}{2} R^2 \sin 2C;$$

likewise, $S_{\Delta BOC} = \dfrac{1}{2}R^2 \sin 2A$, $S_{\Delta COA} = \dfrac{1}{2}R^2 \sin 2B$. Hence

$$S = S_{\Delta AOB} + S_{\Delta BOC} + S_{\Delta COA} = \frac{R^2}{2}(\sin 2A + \sin 2B + \sin 2C)..$$

This equality also holds though the area of one part is zero when ABC is a right triangle.

When ABC is an obtuse triangle, the student assumed $A > \dfrac{\pi}{2}$ without loss of generality so that $S = S_{\Delta AOB} + S_{\Delta COA} - S_{\Delta BOC}$.

Note that $\angle BOC = 2\pi - 2A$. Then

$$S_{\Delta BOC} = \frac{1}{2} R^2 \sin(2\pi - 2A) = -\frac{1}{2} R^2 \sin 2A.$$

Therefore, the conclusion could be derived by imitating the above approach.

When asked how he discovered such an ingenious proof, the student said he was inspired by the derivation of another area formula of triangle $S = S_{\Delta ABC} = \dfrac{1}{2}(a+b+c)r$, where a, b, c are the opposite sides of A, B, C, and r is the radius of the inscribed circle of triangle ABC. He thought of "calculating the total area using the sum of parts," considering that the conclusion was related to the circumcircle radius R, thus triangle ABC could naturally be divided into triangles AOB, BOC and COA.

He was also aware of discussing the situation of an obtuse triangle. After completing his reasoning, he made an analogy of this case with the derivation of the area formula $S = S_{\Delta ABC} = \dfrac{1}{2}(b+c-a)r_a$, where r_a is the radius of the escribed circle of triangle ABC with respect to A.

This solution showed a kind of unexpected yet reasonable artistic charm.

Definitely, many competition problems that have been studied and discussed are still highly "open in solutions." Other solutions or non-trivial relevant problems may also be obtained under the "attack" of new solution seekers (which is of great educational value although less so in academic value).

Meanwhile, there are plenty of explorative problems in competition mathematics, of which the target of problem-solvers is uncertain. As there is no ready method to follow, the solving process often requires thinking and exploration from multiple perspectives.

Therefore, although competition mathematics problems are different in content from open-ended mathematics problems, it is possible to partially achieve the effect of open-ended mathematics problem teaching for the development of gifted students' mathematical ability and creativity by taking advantage of competition mathematics problems.

4. Disputes over China's Competition Mathematics Education

Although quite a few Chinese and foreign mathematicians support and promote mathematics competitions by asserting their educational value, they also hold a very prudent attitude towards these competitions. At the very beginning of the launch of mathematics competitions in China, Luogeng Hua had such a concern: "Will this work (mathematics competition) affect our school education negatively? Will it affect the students' overall development? It might happen if the job is not done properly (Hua, 1956a, p. 2)." Nowadays the involvement of the "general public" and very young children in mathematics competitions have been disputed continuously, which echo Professor Hua's concerns to a certain extent.

In the respect of universalization of competition mathematics education, opinions are widely divided among Chinese scholars. Some argue that competition mathematics education should be oriented to the entire high school student population in order to inspire creative thinking and cultivate problem-solving ability. On the other hand, others argue that competition mathematics education should only apply to a small group of students (e.g., 5%) and should have the scale of their effects controlled. To use an analogy, sports for the general public are for keeping fit, whereas sports practiced by athletes aim to achieve excellency and breakthroughs. Likewise, in terms of the function of popularization, competition mathematics can be targeted at a large population of students. But in terms of talent identification and selection, competition mathematics is only suitable for a small group of students with a strong interest and aptitude for mathematics. Krutetskii (1976) studied three groups of students with different mathematical abilities. Analysis showed that students with mediocre mathematical ability needed more time and efforts than students with strong ability to make mathematical achievements; they tended to feel very troubled when solving new types of mathematical problems and needed assistance to understand general methods; only through rote practice is it possible for them to shorten the reasoning process. Students with strong mathematical ability could work intensively on mathematical activities for a long period of time, without exhibiting any tiredness. On the contrary, mathematically-weak students were prone to feeling more tired after a short period of time in studying mathematics than in studying other subjects. Thus, it can be concluded that it is against the interests of education to require the involvement of too many students in high-level mathematical training.

With regard to the involvement of younger children in mathematics competitions, actually younger-grade award winners are by no means rare in the IMO of previous years. One example is the Fields Medal winner Terence Tao who obtained his IMO gold medal when he was just 12 years old. The launch of mathematics competitions is instrumental for discovering the mathematical gift of such child prodigies. As the eminent mathematician Kolmogorov wrote in his preface to the book 1st -50th Moscow Mathematical Olympiad (Гальперин and Толпыго, 1986): “At the very beginning, the Moscow Mathematical Olympiad was only for Grade 9 and 10 students; from 1940, it started to invite students of Grade 7 and 8. This change in the age of the participants was because students of such grades had already started to show their interest and talent in mathematics.” However, the involvement of overly young students has shortcomings and disadvantages. Kolmogorov further expressed that, “Although it is possible to invite even younger competitors, we could not help but notice that most of these Grade 5 and 6 students who had solved competition problems later lost their problem-solving ability and interest in mathematics as they progressed to higher grades.” It shows that Kolmogorov had his reservations about the involvement of younger participants. According to the research of mathematicians and psychologists, such as V. A. Krutetskii’s tracking study on the 26 gifted children in mathematics (Krutetskii, 1976), M. A. Clements’ case study on the child Terence Tao (Clements, 1984), to name a few, talent in mathematics is formed in early childhood, and gifted children demonstrate a remarkable talent for mathematics and learning speed. So, how to make such students develop their mathematical ability while still maintaining their interest in mathematics is an issue that deserves a great attention when conducting mathematics competitions.

In recent years, as more and younger children are attending competition mathematics lessons, a substantial improvement in problem solving abilities among younger participants has been observed. On the other hand, the difficulty and complexity of the problems have increased and are sometimes out of touch with the classroom teaching. Take the first item in the 2014 National Junior High School Mathematics Competition for example (cf. Xu, 2015):

Example 4.1. Let x, y be integers such that

$$\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x^2}+\frac{1}{y^2}\right)=-\frac{2}{3}\left(\frac{1}{x^4}-\frac{1}{y^4}\right).$$

Then the number of possible values of $x+y$ is (   ).

(A) 1 (B) 2 (C) 3 (D) 4

The essence of the problem is to find out, by observation, that $\frac{1}{x}+\frac{1}{y}$, $\frac{1}{x^2}+\frac{1}{y^2}$ on the left side of the equation are 2 factors of $\frac{1}{x^4}-\frac{1}{y^4}$ on the right side, thus the problem could be simplified; yet after that, a quadratic indeterminate equation should be solved, whether x, y are zero in value also be considered and the case in $\frac{1}{x}+\frac{1}{y}=0$ cannot be neglected. There are too many details to be considered. Moreover, the validity of the option set is also questionable. As the first item in a test, would it not be more advisable to lower the difficulty and try to design problems with the purpose of promoting in-class instruction?

There is an opinion that "when the new contents that appears in mathematics competitions are familiar to and within the grasp of high school students and teachers, the competitions would then have successfully fulfilled their Olympiad mission — to integrate into high school mathematics, or in other words, to have popularized and disseminated mathematical knowledge (Chen and Zhang, 2013, p. 15)." Actually, two challenges are behind the successful popularization. For one, more and more extracurricular contents will be integrated into in-class instruction, resulting in excessive information for students to learn, thus adding to their burden in schoolwork, thereby raising the problem of controlling this tendency. For another, as the contents of competition mathematics become increasingly familiar with the general public, the function of selecting talents may be lost as it becomes more difficult to differentiate between participants, so the cooperation of experts in various fields including mathematicians is required in order to prepare the test problems conscientiously, to ensure the novelty and validity in the problems given. In competition mathematics, we should try to avoid proposing those complicated problems in such fields that are known to all or systematically studied.

In conclusion, China's competition mathematics education serves as a supplement and enhancement for in-class instruction and is also a means to identify and cultivate mathematically gifted students. However, many practices and explorations in the long run are essential for kicking a balance between popularization and selection, as well as the public education and gifted education.

References

C. L. Chen and T. J. Zhang (2013). *A Competition Mathematics Course*. 3rd ed. Beijing: Higher Education Press (in Chinese).

M. A. Clements (1984). Terence Tao. *Educational Studies in Mathematics*, 15(3), 213–238.

Z. P. Dai (2002). *Open-ended Problems: A New Pattern of Mathematics Teaching*. Shanghai: Shanghai Education Press (in Chinese).

Y. F. Feng (2006). *Theory and Practice of Olympiad Mathematics Education*. Shanghai: Shanghai Education Press (in Chinese).

Г.А. Гальперин and А.К. Толпыго (1986). Московские Математические Олимпиады. Москва: Просвещеиии.

L. G. Hua (1956a). Mathematics competition to be held in our country. *Shuxue Tongbao*, (1), 1–2 (in Chinese).

L. G. Hua (1956b). Writing after the mathematics competition in 1956. *Shuxue Tongbao*, (6), 1–2 (in Chinese).

V. A. Krutetskii (1976). *The Psychology of Mathematical Abilities in School Children*. Chicago: University of Chicago Press.

Ministry of Education PRC (2000). Mathematics Syllabus for Full-time Senior High Schools (revised edition). Beijing: People's Education Press (in Chinese).

Ministry of Education PRC (2003). Mathematics Curriculum Standard for Senior High School (experimental). Beijing: People's Education Press (in Chinese).

Ministry of Education PRC (2012). Mathematics Curriculum Standard for Compulsory Education. 2011 Ed. Beijing: Beijing Normal University Press (in Chinese).

National Council of Teachers of Mathematics (2000). Principles and Standards for School Mathematics. Reston, Virginia: NCTM.

National Team Coaching Staff (2014). Towards the IMO: A Collection of Olympiad Mathematics Problems. Shanghai: East China Normal University Press (in Chinese).

Office of High School of REP (2006). Mathematics Review Guide for College Entrance Examinations.cc Beijing: People's Education Press (in Chinese).

Popularization Committee of CMS (2006a). Syllabus for senior high school mathematics competition. 2006 ed. *High School Mathematics*, (12), 3–5 (in Chinese)

Popularization Committee of CMS (2006b). Syllabus for junior high school mathematics competition. 2006 ed. *High School Mathematics*, (12), 2–3 (in Chinese).

Z. R. Sheng and C. Zhou (2010). Review on the researches of international mathematics gifted education. *Journal of Zhejiang Education Institute*, (3), 9–15/60 (in Chinese).

R. Q. Sun and D. T. Hu (1994). *Introduction to Olympiad Mathematics Education*. Beijing: Peking University Press (in Chinese).

S. C. Tang and Z. G. Feng (2011). Early identification and education of mathematics talents: A case study. *Shuxue Tongbao*, 50(3), 11–16 (in Chinese).

B. Xiong and Y. J. He (2012). *Problem Solving Methods and Strategies in Senior High School Competition Mathematics*. Shanghai: East China Normal University Press (in Chinese).

S. L. Xu (2015). National junior high school mathematics competition in 2014, *High School Mathematics*, (1), 21–25 (in Chinese).

H. W. Zhu (2009). *From Mathematics Competition to Competition Mathematics*. Beijing: Science Press (in Chinese)